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1 Introduction

In the period of 2004-2007, the 2D and 3D shallow water flow models Waqua and Triwaq have been attuned to each other and have finally been merged together [2, 3, 4]. During this work, subtle differences between the numerical discretisation schemes have been identified and eliminated. One of these differences concerns the discretisation of the horizontal viscous terms near screens or closed boundaries.

The effect of the integration of Waqua and Triwaq on simulation results was investigated in 2007 by Alkyon [6]. Additional tests were carried out by VORtech early in 2008 [5]. This revealed a large influence of the choice of discretisation of the horizontal viscous terms for several river sections in the Zeedelta model. This influence was attributed to the behaviour of the discretisations near screens/closed boundaries, especially for so-called “staircase boundaries”, where the computational grid is not aligned well with the true geometry.

In [5] it was recommended to study the discretisation of the viscous terms in more detail. This is the purpose of the current report. This work was carried out in change c85419 of the SIMONA B&O project.

2 Set up of our experiments

This report concerns the question which discretisation is better for the horizontal viscous terms:

1. the "present approach", where the boundary condition $u = 0$ is used, or
2. the "Triwaq approach", where the boundary condition $\partial u/\partial n = 0$ is used.

We investigate this question using test models for idealised river sections using grids that are aligned with the river (straight boundaries) or rotated with respect to the river (staircase boundaries).

The so-called Chezy flow problem is used: a stationary situation for a river section with a constant cross section and constant bottom gradient. The flow then becomes constant too, and an analytical solution is available for the water depth and flow velocities as a function of the discharge, the bottom gradient, and the Chezy friction coefficient.

We identify how well the analytical solution is reproduced by different test models and discretisation methods used. Ideally the analytical solution is retrieved, and this appears to be the case for the test model with straight boundaries. In this case the viscosity terms do not cause additional resistance for the flow, which corresponds to the desired "free slip" boundary. For the rotated test models deviations from the analytical solutions are obtained. The water depth becomes higher than the desired result. This is caused by additional resistance in the models and formulations that are used. This additional resistance will appear to be substantially higher for the present than for the Triwaq approach.

3 Analytic solution of the Chezy model

The differential equations for shallow water flow are given in the technical documentation of Waqua/Triwaq [1] in Equations (2.116) and (2.118). We consider a river section that is aligned with the y-direction. In the stationary situation the solution will be uniform in the x-direction. Therefore we ignore the u-momentum equation and all dependence on x. The equations in the y-direction are given by:

$$\zeta_t + (Hv)_y = 0, \tag{1}$$

$$v_t + vv_y + g\zeta_y - \nu_h v_{yy} + \frac{gv|v|}{C^2 H} = 0. \tag{2}$$

In this equation ζ is the water level, H is the water depth, g the gravity constant, ν_h the horizontal viscosity parameter and C the Chezy coefficient.

In this analysis we consider a stationary situation, which means that the derivatives with respect to time in (1) and (2) are dropped. We further consider a situation in which the parameters C, ν_h and the bottom gradient i_b are independent of the position y. We seek solutions that are independent of position y too.

From equation (1) we conclude that the discharge Hv is constant in space and in time. Assuming that v is constant too leads to a constant water depth H and gives

$$\zeta_y := i_b. \tag{3}$$

Substituting this into (2) and dropping the zero derivatives of v yields

$$g\ i_b + \frac{gv|v|}{C^2 H} = 0. \tag{4}$$

In the case that the bathymetry gradient is positive ($i_b > 0$), v must be negative ($v < 0$) and $|v| = -v > 0$. In this case we can rewrite Equation (4) and we end up with the following solution for v:

$$v = -\sqrt{i_b C^2 H}. \tag{5}$$

In the case that the bathymetry gradient is negative, v must be positive and $|v| = v > 0$. In this case we can rewrite Equation (4) as:

$$v = \sqrt{-i_b C^2 H}. \tag{6}$$

If we know the width B of the river and the discharge Q upstream we can calculate the corresponding water depth H by using that $Q = vBH$ and substituting Equation (5) (or (6)). We can then rewrite the equations such that we have H and v as a function of Q. In this case we end up with:

$$H = \sqrt[3]{Q^2/i_b C^2}, \quad v = Q/BH. \tag{7}$$

This analytical solution is used for the definition of our test problems. In the Waqua-input we impose a constant discharge Q at the upstream boundary and the corresponding water depth H at the downstream boundary. If the model is simulated well, the stationary end situation should comply with the analytical results. Otherwise, inaccuracies in the discretisation have been found.

4 Determining the dimensions for our Chezy model

We want to make a Chezy-flow model which is representative for the Zeedelta-model since this model is the motivation for this research. Using this model we try to choose the needed parameters for the test model. We need the with (B) of the channel, the bathymetry gradient (i_b), a water depth (H) at outflow, a capacity (Q) at inflow, a Chezy coefficient (C) and representative choices for Δx and Δy.

If we look at Figure 1 we can see that the width of the river has an average of about 250 meters. Furthermore there are about 13 grid cells in that direction. So in our Chezy-flow model we will use $\Delta x = 20$ meters. Furthermore in the Zeedelta model about 8 grid cells are used per kilometer in the flow direction. In our test model we will take $\Delta y = 120$ meters.

In Figure 1 we also can see that the bottom depth of the river varies from about 3.5 to 7 meters in a distance of approximately 10 kilometer. We choose the bottom gradient to be $i_b = 3.6 \cdot 10^{-4}$. Now since $\Delta y = 120$, the bottom varies with 4.32 cm per grid cell. Upstream we choose a depth of 4.5 meters, so downstream we have a bathymetry of 5.796 meters $(4.5 + 30 \cdot 0.0432)$.

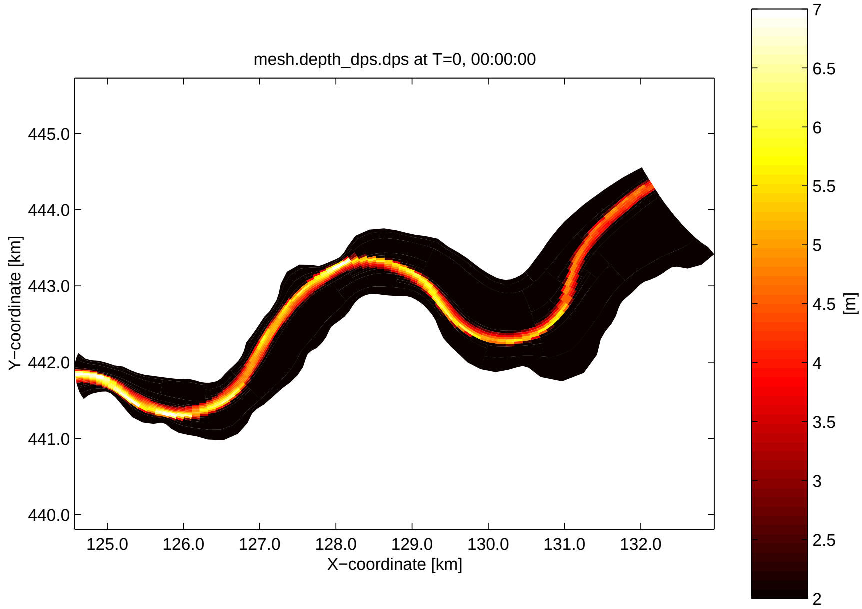

Figure 1: *Bottom depth in a river section (Lek) of the Zeedelta model.*

To find the Chezy values we have to do a little more work. The Zeedelta model does not use Chezy values for the bottom roughness, but uses the White-Colebrook formulation instead. Fortunately we can calculate the corresponding Chezy values. The values for the Zeedelta model are stored on the SDS-file as `czu` and `czv` (for u and v direction). In the case of the Zeedelta model these values are the same and we will call them `cz`. Now `cz` can be calculated using Equation (7.58) of [1]:

$$\texttt{cz} := \frac{1}{2}\frac{g}{C^2}\Delta t, \quad C := \sqrt{\frac{1}{2}\frac{g}{\texttt{cz}}\Delta t}. \tag{8}$$

Here C is the Chezy value we are looking for. The results for the Zeedelta model are given in Figure 2. In this figure we can see that the Chezy parameter inside the river is not varying much and for our Chezy-flow model we will take a constant value of 50.

For the Chezy-flow model we need a water depth. If we choose a value for H we can use Equation (5) to calculate the corresponding v. After that we can calculate the corresponding value for Q upstream using $Q = vBH$ in order to get a stationary flow with a constant water depth. If we look at Figure 3 we see that inside the river we have a water depth of approximately 5.5 meters. For computational reasons we choose $H = 5.6250$ [m] and we end up with $v = 2.25$ [ms^{-1}].

5 Setting up test models

In the previous section we identified parameters that characterize a section of the river Lek in the Zeedelta model. These parameters are the following:

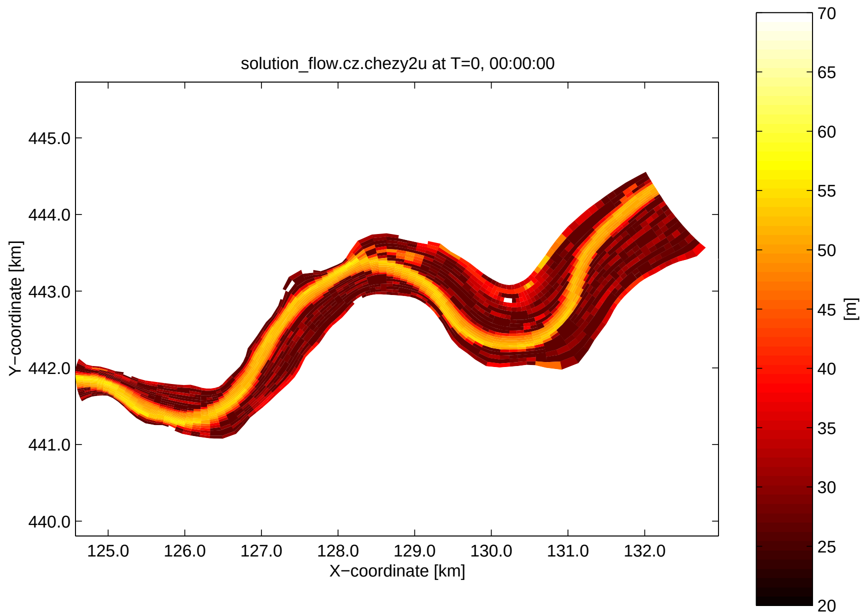

Figure 2: *Chezy values of the Zeedelta model.*

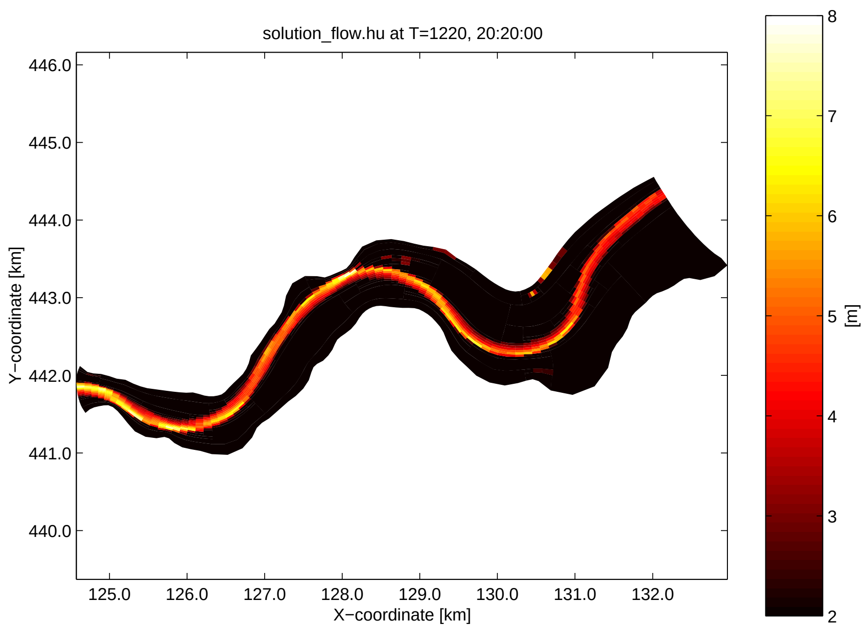

Figure 3: *Water depth H in the Zeedelta model.*

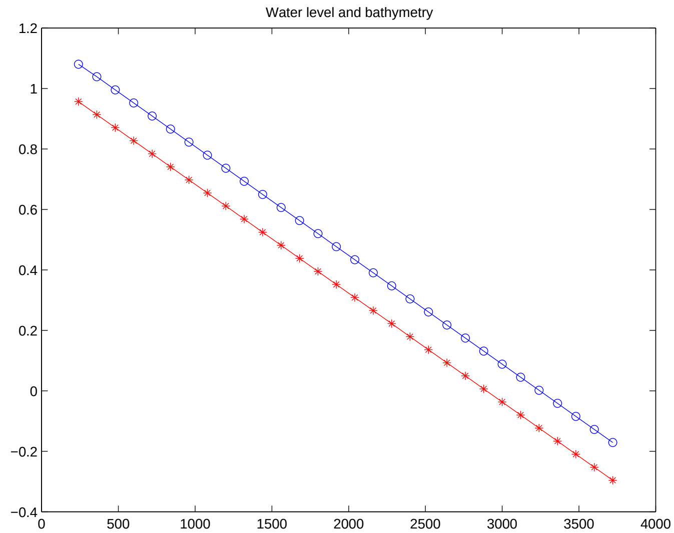

Figure 4: *Computed water levels (blue) and bottom depths (red, shifted upwards by 5.5 m) along the center line of the river section in test model 1, using a straight grid.*

- $B = 260$ $[m]$,
- $i_b = 3.6 \cdot 10^{-4}$ [-],
- $C = 50$ $[m^{\frac{1}{2}} s^{-1}]$,
- $Q = 253.125$ $[m^3 s^{-1}]$,
- $v = 2.25$ $[ms^{-1}]$,
- $H = 5.6250$ $[m]$.

We model a river section of 3.6 km using three different grids:

1. straight grid: grid lines ξ, η aligned with physical coordinates x, y; step sizes $\Delta x = 20m, \Delta y = 120m$;
2. rotated grid, 4.8°; step sizes $\Delta\xi = 10m, \Delta\eta = 120m$; the boundary makes a step sideways (ξ/m-direction) in each forward step (η/n-direction), 1:1 ratio;
3. rotated grid, 4.8°; step sizes $\Delta\xi = 10m, \Delta\eta = 20m$; the boundary steps sideways in every sixth forward step.

First we want to be sure that we calculated all the parameters in the right way. For this the first test case with a straight grid is used. In this test the closed boundaries have no effect on the results. The results for this test case are shown in Figure 4. This figure shows the computed water levels ζ in a stationary condition (blue circles) and the bottom depths (red

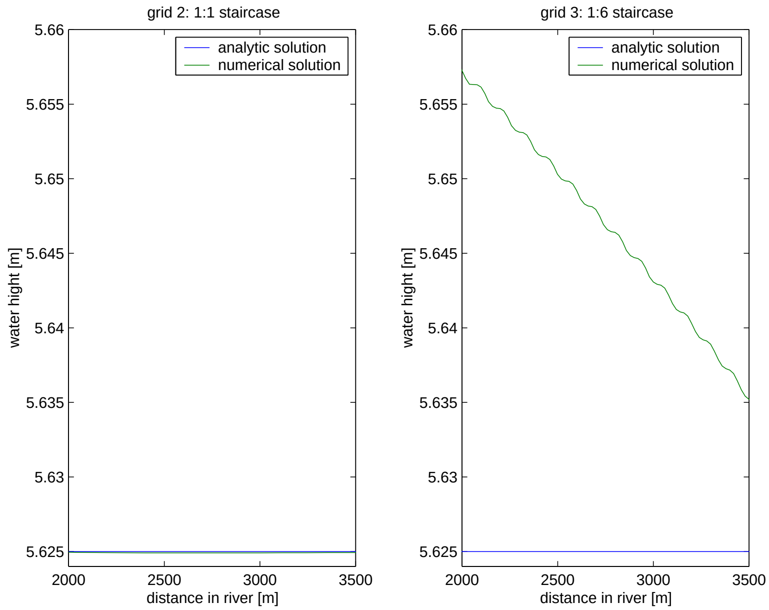

Figure 5: *Water depths along the centerline of the river section for the rotated grids for a situation* without *horizontal viscosity,* $\nu_h = 0$*. Left: grid 2, full staircase boundary, right: grid 3, sideways step at every sixth forward step.*

stars, shifted upward 5.5 meters in order to fit into the graph). This figure illustrates that the analytical solution is perfectly retrieved when using a well aligned grid.

6 Results

In this section we present our results with respect to the influence of the horizontal viscous terms for rotated grids.

Before we start investigating the effect of horizontal viscosity, we present results for test runs without viscosity, with $\nu_h = 0$. Figure 5 shows the results for grids 2 (left) and 3 (right). This figure shows that the analytic solution is perfectly retrieved for grid 2, whereas large errors are observed for grid 3. The source of these errors is illustrated in Figure 6. This figure shows that the flow is hindered much by the steps in the side walls. The flow appears to be parallel to the right side wall for five grid cells and then turns sharply to the left. A rather large sideways velocity is needed for this, which is accomplished by a locally large waterlevel gradient.

These initial results show that we can obtain good solutions using rotated grids, but only when the staircase boundary shifts sideways in every forward step.

Now we compare the same four test cases for a situation *with* viscosity, with a rather large value $\nu_h = 6m^2/s$. The resulting waterlevels are shown in Figure 7. A further comparison between the different test cases is presented in Table 1.

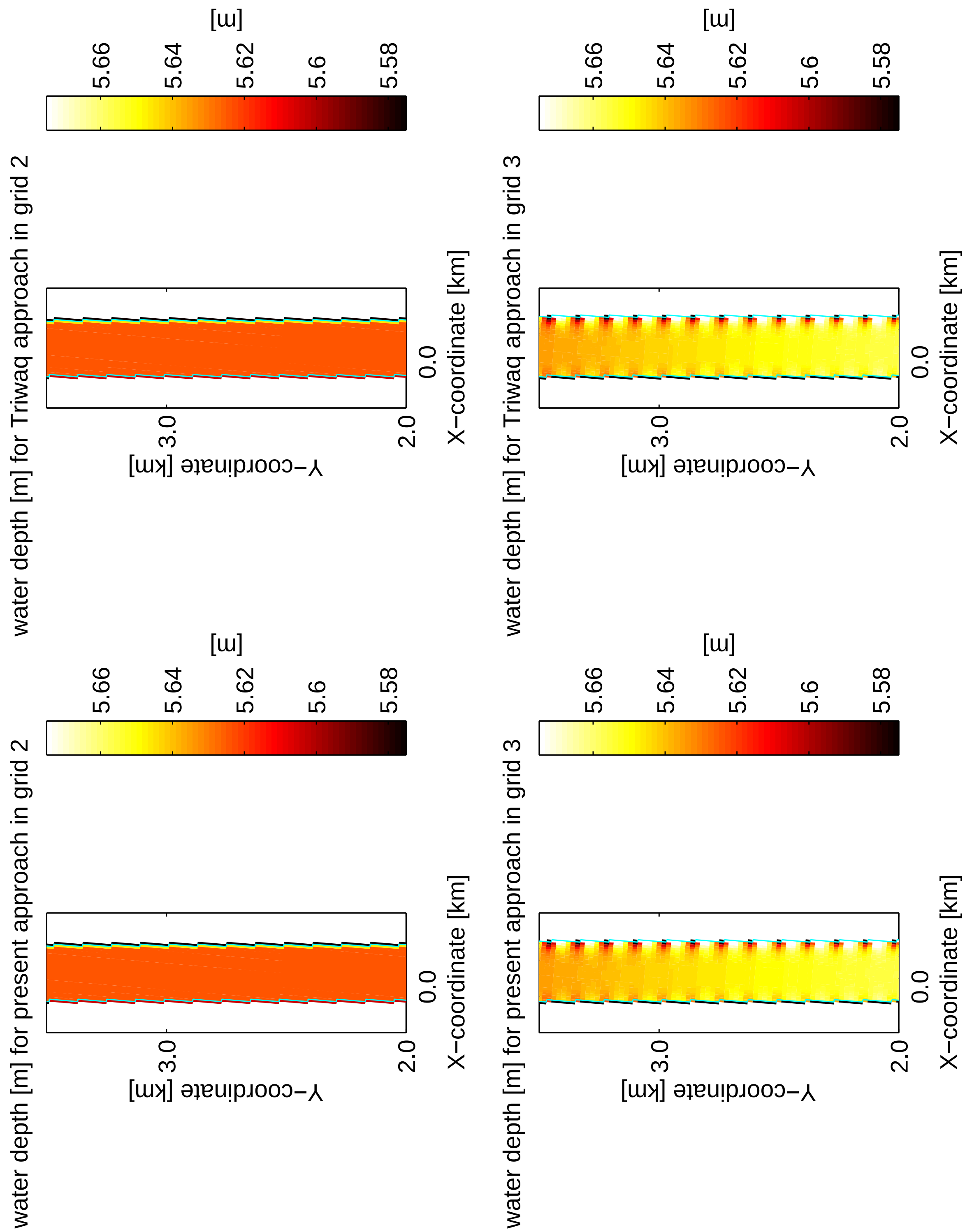

Figure 6: *Computed waterlevels near the downstream boundary for grids 2 (top) and 3 (bottom), using the present approach (left) and Triwaq approach for viscosity (right). In this case the approaches give identical results because no viscosity is used,* $\nu_h = 0$.

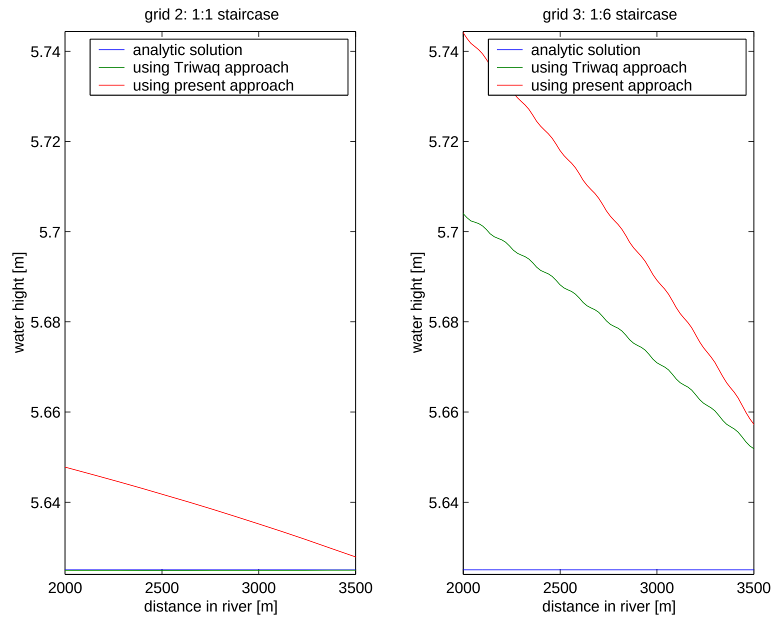

Figure 7: *Water depths along the centerline of the river section for the rotated grids for a situation* with *horizontal viscosity,* $\nu_h = 6m^2/s$*. Left: grid 2, full staircase boundary, right: grid 3, sideways step at every sixth forward step.*

Figure 7 illustrates that adding viscosity degrades the results for the present approach. Differences between the computed and analytical solutions are introduced that were not present in Figure 5. The Triwaq approach displays better results. It still functions correctly for grid 2 (Figure 7, left), and for grid 3 the additional errors are smaller than for the present approach.

In table 1 we look at the difference in the water level between the numerical and the analytical solution near the center point of the canal at $y = 2\,km$. In this way we diminish local effects of boundary conditions. Also we only look at the middle of the river. We can see that grid 2 performs better than grid 3 and that the Triwaq approach is more exact than the present approach.

7 Implementation in SIMONA

Based on the results of the previous section, presented to Deltares in version 0.3 of this report, it was decided to implement the "Triwaq approach" as the new default method in Waqua/Triwaq in SIMONA. The "present approach" is provided too, for reasons of backwards compatibility. For this the keyword "OLD_BND_TREATM" is added in the simulation input file, in the section "FLOW - PROBLEM - VISCOSITY". When this keyword is provided the "present approach" is used, if it is omitted the "Triwaq approach" is used instead.

This choice for the boundary treatment for the viscosity term affects the simulation results. The SIMONA test bank is used to assess the differences.

	grid 2, full staircase		grid 3, 1:6 staircase	
	present approach	Triwaq approach	present approach	Triwaq approach
Visc = 6.0	0.026	-0.000060	0.129	0.085
Visc = 5.0	0.022	-0.000052	0.114	0.079
Visc = 4.0	0.017	-0.000042	0.099	0.073
Visc = 3.0	0.012	-0.000028	0.085	0.067
Visc = 2.0	0.0075	-0.000017	0.070	0.060
Visc = 1.0	0.0033	0.000010	0.055	0.052
Visc = 0.1	0.00028	0.000058	0.038	0.037
Visc = 0.0	0.000072	0.000072	0.035	0.035

Table 1: *Differences between numerical and analytical solution using two different formulations [m].*

	Waterlevel $[m]$		Velocity $[ms^{-1}]$	
Model	difference	sensitivity	difference	sensitivity
`restart1.waq`	0.112594	0.000755	0.151389	0.000587
`integral`	0.112594	0.000002	0.151394	0.000003

Table 2: *Largest and most striking differences between the "present" and "Triwaq" approaches for the models of the SIMONA test bank.*

The differences are somewhat larger than for most changes to the model code. The 99% largest differences in waterlevels reach up to about $10\,cm$ for several models. Whether this is correct or not is decided by comparing the differences to those that were found in the uniformation of Waqua and Triwaq, which are listed in [3].

If we compare our differences with the differences listed in [3], two models attract our attention. These models are `integral` and `restart1.waq`. The differences in these models are much bigger now than the differences due the uniformation project. These results can be found in Table 2.

The reason why the differences are now so much bigger than in the uniformation project is that these test cases concern Waqua models. For Triwaq models, the viscosity was changed in the uniformation project and now we change it back. For Waqua models, the viscosity was not changed in the uniformation project, such that we may expect larger differences in the current work.

Another reason why the testmodels `restart1.waq` and `integral` show such large differences is that they both concern variants of the KTV model. This model has a coarse resolution and contains many staircase boundaries. Therefore it is highly sensitive to the boundary treatment for the viscosity terms.

8 Conclusions

In this report we have analysed the influence of horizontal viscous terms on the ability of Waqua/Triwaq to simulate the Chezy flow problem for rotated grids, grids that are not aligned well with the actual river geometry.

Two different test cases were compared: a grid schematisation with stretched grid cells, where the closed boundary of the model domain steps sideways at every forward step, and a grid schematisation with square grid cells where the sideways steps occur at every sixth forward step. Two different discretisation schemes were compared for the viscous terms too: the "present approach" which is used in both Waqua and Triwaq in SIMONA release 2008-01, and the "Triwaq approach" that was used in Triwaq in version 2007-01 and earlier.

The results show that the Triwaq approach for the horizontal viscous terms is better in the sense that it introduces less resistance at closed boundaries. Consequently it was decided to implement the Triwaq approach in Waqua/Triwaq. In order to be able to reproduce current model results the present approach is be made available too, using a switch in the simulation input file. After an adaptation period of one or two years this switch should be removed from the code.

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